Nome: .... **RA:** ....

Obs.: Resolva as questões nas folhas de papel almaço e copie o resultado no espaço apropriado.

1	
2	
3	
4	
5	
6	
7	
8	

1ª Questão: Determine a resposta ao impulso do sistema

$$y[n] = \sum_{k=-\infty}^{\infty} k \left(\frac{1}{2}\right)^{-k} x[n-k]u[k]$$

e utilize-a para classificá-lo quanto à linearidade, invariância no tempo, BIBO estabilidade e causalidade. (1,0 ponto)

2ª Questão: Sabendo que a transformada Z do sinal discreto $x[n]$ é dada por

$$\mathcal{Z}\{x[n]\} = X(z) = \frac{27z^2}{9z^2-1}, \ |z| > 1/3,$$

determine: (2,0 pontos)

a)$x[0]$ b)$x[\infty]$ c) $\sum_{n=-\infty}^{\infty} x[n]$ d)$x[n]$

3ª Questão: Seja $\mathbb{W} = 2\mathbb{X} + \mathbb{Y}$ tal que $\mathbb{X}$ e $\mathbb{Y}$ são variáveis aleatórias discretas independentes com transformadas Z respectivamente dadas por

$$G_{\mathbb{X}}(z) = \frac{5}{4-z}, \ |z| < 2 \quad \text{e} \quad G_{\mathbb{Y}}(z) = \frac{2}{5-z}, \ |z| < 5.$$

Calcule: (1,0 ponto)
a) A distribuição de probabilidades da variável aleatória $\mathbb{W}$ em $k = 1$, ou seja, $p[1] = \Pr\{\mathbb{W} = 1\}$.
b) A média da variável aleatória $\mathbb{W}$, ou seja, $\bar{w} = \mathcal{E}\{\mathbb{W}\}$.

4ª Questão: Considere o sinal discreto periódico $x[n] = 1 + 2\exp(j\pi n) - 4\cos\left(\frac{4\pi}{3}n\right) + j6\text{sen}\left(\frac{\pi}{2}n\right)$.

a) Determine o período fundamental N de $x[n]$. (0,5 ponto)

b) Determine os coeficientes c_k de $k = 0, \dots, N-1$ da série exponencial de Fourier. (0,5 ponto)

c) Determine a potência média de $x[n]$. (0,5 ponto)

5ª Questão: Determine e esboce $x(t) * y(t)$, para $x(t) = G_2(t)$ e $y(t) = 2G_1(t+0.5) - G_2(t-1)$ (1,0 ponto)

6ª Questão: a) Determine os coeficientes c_k da série exponencial de Fourier de (0,5 ponto)

$$x(t) = \sum_{k=-\infty}^{\infty} p(t-6k), \quad p(t) = t\ G_1(t-0.5).$$

b) Calcule c_0. (0,5 ponto)

c) Determine a potência média de $x(t)$. (0,5 ponto)

$7^{\underline{a}}$ Questão: a) Determine o valor da integral $I = \int_{-\infty}^{\infty} x(t)\text{Sa}(t)dt$ com $x(t) = \frac{d^2}{dt^2}\left(-\frac{2}{\pi}\text{Sa}(2t)\right)$. (1,0 ponto)

$8^{\underline{a}}$ Questão: Sabendo que a transformada de Laplace de $x(t)$ é dada por

$$\mathcal{L}\{x(t)\} = X(s) = \frac{3-9s}{s^2-4s-5},\ -1 < \text{Re}(s) < 5,$$

determine: (1,0 ponto)

a) $\int_{-\infty}^{+\infty} tx(t)dt$ b)$x(t)$

Convolução de Sinais discretos: $x_1[n] * x_2[n] = \sum_{k=-\infty}^{+\infty} x_1[k]x_2[n-k]$, $x[n] * \delta[n] = x[n]$, $x[n] * \delta[n-m] = x[n-m]$

Sistemas Lineares Invariantes no Tempo (SLIT):

$$\Rightarrow \; y[n] = x[n]*h[n] \;,\; h[n] = \mathcal{G}\{\delta[n]\} \;,\; y[n] = z^n * h[n] = H(z)z^n \;,\; H(z) = \sum_{k=-\infty}^{+\infty} h[k]z^{-k} = \mathcal{Z}\{h[n]\}$$

Transformada Z:

$$\mathcal{Z}\{x[n]\} = \sum_{k=-\infty}^{+\infty} x[k]z^{-k}, \quad \mathcal{Z}\{a^n u[n]\} = \frac{z}{z-a} \;,\; |z| > |a| \quad , \quad \mathcal{Z}\{-a^n u[-n-1]\} = \frac{z}{z-a} \;,\; |z| < |a|$$

$$\mathcal{Z}\{na^{n-1}u[n]\} = \frac{z}{(z-a)^2} \;,\; |z| > |a| \quad , \quad \mathcal{Z}\{-na^{n-1}u[-n]\} = \frac{z}{(z-a)^2} \;,\; |z| < |a|$$

$$\mathcal{Z}\{x[n]\} = X(z), z \in \Omega_x \;\Leftrightarrow\; \mathcal{Z}\{x[-n]\} = X(z^{-1}), z^{-1} \in \Omega_x \quad , \quad \mathcal{Z}\{x_1[n] * x_2[n]\} = \mathcal{Z}\{x_1[n]\}\mathcal{Z}\{x_2[n]\}$$

$$\mathcal{Z}\{n^m x[n]\} = \left(-z\frac{d}{dz}\right)^m X(z) \quad , \quad \sum_{k=-\infty}^{+\infty} k^m x[k] = \mathcal{Z}\{n^m x[n]\}\Big|_{z=1} \;,\; 1 \in \Omega_x \;,\; m \in \mathbb{N}$$

$$\mathcal{Z}\{y[n] = x[n-m]u[n-m]\} = z^{-m}\mathcal{Z}\{x[n]u[n]\} \quad , \quad m \in \mathbb{Z}_+ \;,\; \Omega_y = \Omega_x$$

$$\mathcal{Z}\{x[n+m]u[n]\} = z^m\Big(\mathcal{Z}\{x[n]u[n]\} - \sum_{k=0}^{m-1} x[k]z^{-k}\Big) \;,\; m \in \mathbb{Z}_+$$

$$\mathcal{Z}\left\{\begin{pmatrix} n \\ m \end{pmatrix} a^{n-m}u[n]\right\} = \frac{z}{(z-a)^{m+1}} \;,\; |z| > |a| \;,\; m \in \mathbb{N} \quad , \quad \mathcal{Z}\{na^n u[n]\} = \frac{az}{(z-a)^2} \;,\; |z| > |a|$$

$$\mathcal{Z}\left\{\begin{pmatrix} n+m \\ m \end{pmatrix} a^n u[n]\right\} = (1 - az^{-1})^{-(m+1)} = \frac{z^{m+1}}{(z-a)^{m+1}} \;,\; m \in \mathbb{N} \;,\; |z| > |a|$$

$$x[0] = \lim_{|z|\to+\infty} X(z) \;,\; \Omega_x \text{ exterior de um círculo} \quad , \quad x[+\infty] = \lim_{z\to1}(z-1)X(z) \;,\; |z| > \rho \;,\; 0 < \rho \leq 1$$

Transformada Z aplicada à probabilidade:

$$G_{\mathbb{X}}(z) = \mathcal{E}\{z^{\mathbb{X}}\} = \mathcal{Z}\{p[n]\} = \sum_{k=-\infty}^{+\infty} p[k]z^k = \sum_{k=-\infty}^{+\infty} \Pr\{\mathbb{X} = k\}z^k$$

Seqüências $p[n]$ à direita do 0: $G_{\mathbb{X}}(z) = \sum_{n=0}^{+\infty} \frac{1}{n!}\frac{d^n}{dz^n}G_{\mathbb{X}}(z)\Big|_{z=0} z^n \quad \Rightarrow \quad p[n] = \frac{1}{n!}\frac{d^n}{dz^n}G_{\mathbb{X}}(z)\Big|_{z=0}$

$$\mathcal{E}\{\mathbb{X}\} = \sum_k kp[k] \quad , \quad \sigma_{\mathbb{X}}^2 = \mathcal{E}\{\mathbb{X}^2\} - \mathcal{E}\{\mathbb{X}\}^2 \quad , \quad \mathcal{E}\{\mathbb{X}^m\} = \left(\frac{zd}{dz}\right)^m \mathcal{Z}\{p[n]\}\Big|_{z=1}$$

$\mathbb{X}$, $\mathbb{Y}$ var. aleatórias independentes $\quad \Rightarrow \quad G_{a\mathbb{X}+b\mathbb{Y}} = \mathcal{E}\{z^{(a\mathbb{X}+b\mathbb{Y})}\} = \mathcal{E}\{z^{a\mathbb{X}}\}\mathcal{E}\{z^{b\mathbb{Y}}\} = G_{\mathbb{X}}(z^a)G_{\mathbb{Y}}(z^b)$

Série de Fourier de sinais discretos:

$x[n] = \exp(j\beta n)$ periódica $\Leftrightarrow \beta = 2\pi\frac{p}{q}, \;\; p, q \in \mathbb{Z}$ Potência média: $\frac{1}{N}\sum_{n\in\bar{N}} |x[n]|^2 = \sum_{k\in\bar{N}} |c_k|^2$

$x[n] = \sum_{k\in\bar{N}} c_k \exp\Big(jk\frac{2\pi}{N}n\Big) \;,\; c_k = \frac{1}{N}\sum_{n\in\bar{N}} x[n]\exp\Big(-jk\frac{2\pi}{N}n\Big)$, $\bar{N}$ conj. de N inteiros consecutivos

Sinais Contínuos e convolução:

$G_T(t) = u(t+\frac{T}{2}) - u(t-\frac{T}{2})$, $\text{Tri}_2(t) = (t+1)G_1(t+\frac{1}{2}) + (1-t)G_1(t-\frac{1}{2})$, $\int_{-\infty}^{+\infty} f(t)\delta(t)dt = f(0)$

$$\delta(t) = \frac{d}{dt}u(t),\ x_1(t)*x_2(t) = \int_{-\infty}^{\infty} x_1(\beta)x_2(t-\beta)d\beta, \quad x(t)*\delta(t) = x(t), \quad x(t)*u(t) = \mathcal{I}_x(t) = \int_{-\infty}^{t} x(\beta)d\beta$$

$$\mathcal{I}_{x*y}(t) = x(t)*\mathcal{I}_y(t) = \mathcal{I}_x(t)*y(t) = u(t)*x(t)*y(t)\ , \quad \frac{d}{dt}\big(x(t)*y(t)\big) = \dot{x}(t)*y(t) = x(t)*\dot{y}(t)$$

$$\mathcal{L}\{\exp(-at)u(t)\} = \frac{1}{s+a},\ \text{Re}(s+a) > 0, \quad \mathcal{L}\{y(t) = x(t-\tau)\} = X(s)\exp(-s\tau),\ \Omega_y = \Omega_x$$

Série de Fourier de Sinais Contínuos:

$$x(t) = \sum_{k=-\infty}^{+\infty} c_k \exp(jk\omega_0 t) \Leftrightarrow c_k = \frac{1}{T}\int_T x(t)\exp(-jk\omega_0 t)dt\ , \quad \frac{1}{T}\int_T |x(t)|^2 dt = \sum_{k=-\infty}^{+\infty} |c_k|^2 \text{ (potência média)}$$

$$\mathcal{F}_S\{x(t)\}_T = \{c_k\}_{\omega_0} \quad \Rightarrow \quad c_0 = \frac{1}{T}\int_T x(t)dt \text{ (valor médio)}\ ,\ x(0) = \sum_{k=-\infty}^{+\infty} c_k$$

$$\mathcal{F}_S\Big\{\frac{d}{dt}x(t)\Big\}_T = \{jk\omega_0 c_k\}_{\omega_0}\ , \quad \mathcal{F}_S\Big\{\int_{-\infty}^{t} x(\beta)d\beta\Big\}_T = \Big\{\frac{1}{jk\omega_0}c_k\Big\}_{\omega_0} (x(t) \text{ com valor médio } 0)$$

Transformada de Fourier de Sinais Contínuos:

$$X(\omega) = \mathcal{F}\{x(t)\} = \int_{-\infty}^{+\infty} x(t)\exp(-j\omega t)dt, \quad x(t) = \mathcal{F}^{-1}\{X(\omega)\} = \frac{1}{2\pi}\int_{-\infty}^{+\infty} X(\omega)\exp(j\omega t)d\omega$$

$$\int_{-\infty}^{+\infty} |\,x(t)|^2 dt = \frac{1}{2\pi}\int_{-\infty}^{+\infty} |\,X(\omega)|^2 d\omega\ , \qquad \mathcal{F}\{x(t)\} = X(\omega) \Leftrightarrow \mathcal{F}\{X(t)\} = 2\pi x(-\omega)$$

$$\mathcal{F}\{G_T(t)\} = T\text{Sa}(\omega T/2), \quad \text{Sa}(x) = \frac{\text{sen}(x)}{x}, \quad \mathcal{F}\{\text{Sa}(\omega_0 t/2)\} = \frac{2\pi}{\omega_0}G_{\omega_0}(\omega), \quad \mathcal{F}\{\text{Sa}^2(\omega_0 t/2)\} = \frac{2\pi}{\omega_0}\text{Tri}_{2\omega_0}(\omega),$$

$$\mathcal{F}\{\text{Tri}_{2T}(t)\} = T\text{Sa}^2(\omega T/2),\ \text{Tri}_{2T}(t) = \frac{1}{T}G_T(t)*G_T(t), \quad \mathcal{F}\{\delta(t)\} = 1,\ \mathcal{F}\{1\} = 2\pi\delta(\omega),$$

$$\mathcal{F}\{u(t)\} = \pi\delta(\omega) + \frac{1}{j\omega}, \quad \mathcal{F}\Big\{\mathcal{I}_x(t) = \int_{-\infty}^{t} x(\beta)d\beta\Big\} = X(\omega)\Big(\pi\delta(\omega) + \frac{1}{j\omega}\Big), \quad \mathcal{F}\{\text{sinal}(t)\} = \frac{2}{j\omega},$$

$$\mathcal{F}\{\exp(-a|t|)\} = \frac{2a}{a^2+\omega^2},\ a > 0,\ \mathcal{F}\{x(t-\tau)\} = X(\omega)\exp(-j\omega\tau),\ \mathcal{F}\{x(-t)\} = X(-\omega)$$

$$\mathcal{F}\{\delta(t-\tau)\} = \exp(-j\omega\tau),\ \mathcal{F}\{x(t)\exp(j\omega_0 t)\} = X(\omega-\omega_0),\ \mathcal{F}\big\{x(t)*y(t)\big\} = X(\omega)Y(\omega)$$

$$\mathcal{F}\{\cos(\omega_0 t)\} = \pi\delta(\omega-\omega_0) + \pi\delta(\omega+\omega_0), \quad \mathcal{F}\{\text{sen}(\omega_0 t)\} = \frac{\pi}{j}\delta(\omega-\omega_0) - \frac{\pi}{j}\delta(\omega+\omega_0)$$

$$\mathcal{F}\{\sum_{k=-\infty}^{+\infty}\delta(t-kT)\} = \omega_0\sum_{k=-\infty}^{+\infty}\delta(\omega-k\omega_0), \quad \omega_0 = \frac{2\pi}{T}, \quad \mathcal{F}\Big\{\frac{d}{dt}x(t)\Big\} = (j\omega)X(\omega)\ ,$$

$$\mathcal{F}\{x(t)y(t)\} = \frac{1}{2\pi}X(\omega)*Y(\omega) \quad , \quad \mathcal{F}\{t^m x(t)\} = j^m\frac{d^m}{d\omega^m}X(\omega)$$

Transformada de Laplace:

$$H(s) = \mathcal{L}\{h(t)\} = \int_{-\infty}^{+\infty} h(t)\exp(-st)dt\ , \quad s \in \Omega_h\ , \quad \int_{-\infty}^{+\infty} x(t)dt = X(s)\Big|_{s=0,\ 0\in\Omega_x}$$

$$\mathcal{L}\{\delta(t)\} = 1,\ s \in \mathbb{C}\ ,\ \mathcal{L}\{x(t) = x_1(t)*x_2(t)\} = \mathcal{L}\{x_1(t)\}\mathcal{L}\{x_2(t)\}\ , \quad \Omega_x = \Omega_{x_1}\cap\Omega_{x_2}$$

$$\mathcal{L}\{y(t) = x(t-\tau)\} = X(s)\exp(-s\tau)\ , \quad \Omega_y = \Omega_x\ , \quad \mathcal{L}\{\exp(-at)u(t)\} = \frac{1}{s+a}\ , \quad \text{Re}(s+a) > 0$$

$$\mathcal{L}\{\exp(-\alpha t)\cos(\beta t)u(t)\} = \frac{(s+\alpha)}{(s+\alpha)^2+\beta^2}\ , \mathcal{L}\{\exp(-\alpha t)\text{sen}(\beta t)u(t)\} = \frac{\beta}{(s+\alpha)^2+\beta^2}\ ,\ \text{Re}(s+\alpha) > 0$$

$$\mathcal{L}\{\exp(-at)x(t)\} = X(s+a),\ (s+a) \in \Omega_x; \quad \mathcal{L}\Big\{\frac{t^m}{m!}\exp(-at)u(t)\Big\} = \frac{1}{(s+a)^{m+1}},\ \text{Re}(s) > -a, m \in \mathbb{N}$$

$$\mathcal{L}\Big\{y(t) = \int_{-\infty}^{t} x(\beta)u(\beta)d\beta\Big\} = \frac{1}{s}\mathcal{L}\{x(t)\}\ , \quad \Omega_y \supset \Omega_x \cap \{s \in \mathbb{C} : \text{Re}(s) > 0\}$$

$$\mathcal{L}\Big\{\frac{t^m}{m!}u(t)\Big\} = \frac{1}{s^{m+1}}\ , \quad \text{Re}(s) > 0\ ,\ m \in \mathbb{N}\ , \quad \mathcal{L}\{x(-t)\} = X(-s)\ , \quad -s \in \Omega_x$$

$$\mathcal{L}\{y(t) = t^m x(t)\} = (-1)^m\frac{d^m X(s)}{ds^m}\ , \quad \Omega_y = \Omega_x\ ,\ m \in \mathbb{N}\ , \quad \mathcal{L}\{\dot{x}(t)\} = sX(s)\ , \quad \Omega_{\dot{x}} \supset \Omega_x$$